Framework for design space reduction in early stage design under uncertainty

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The impact of decisions made under uncertainty can be measured using regret and complexity

Regret is a metric that measures the sub-optimality of a design compared to the true optimal design. Design margins that are added to account for uncertainty results in designs that are sub-optimal for the realized uncertainty values. Complexity is a metric that measures the size of a reduced design space. This is important because the size of a design space affects the computation time to explore the space.

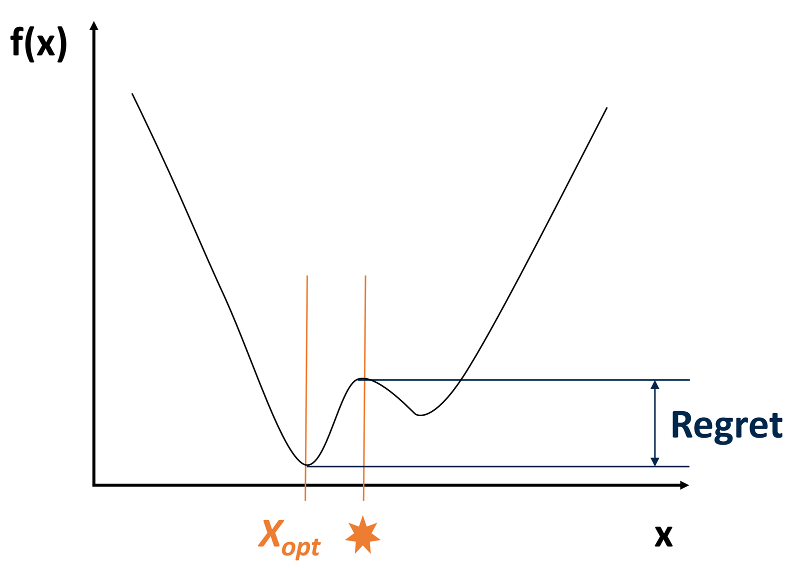

Figure 2: The regret for the selection of the stared point is shown given the function $f(x)$ and the true optimal solution.

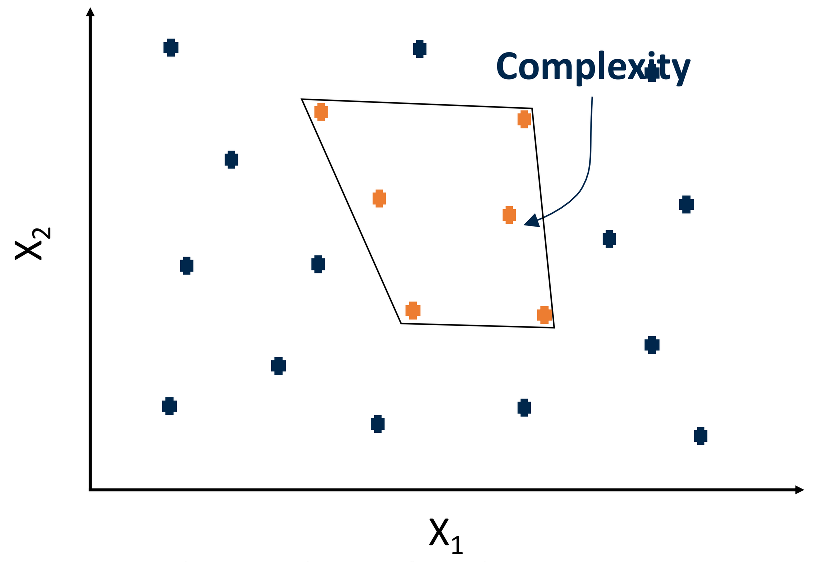

Figure 3: The complexity of a sub-space is a metric for measuring the size of the subspace compared to the original design space.

Complexity and regret were calculated as :

$$F_R(\bar{x}) = \sum_{i=1}^{n} \frac{f(\bar{x}^i) - f(\tilde{x}^i)}{n}$$

$$F_C(\bar{x}) = \prod_{j=1}^{n} s_j$$

$$s_j = \sum_{i=1}^{k_j} \begin{cases} 1 & \text{if } \boldsymbol{x}_j^i \text{ is in } \bar{x}_j \\ 0 & \text{otherwise} \end{cases}$$

The original design space is divided into two spaces, one for design decision parameters, and one for uncertain parameters,

The objective function $f(z)$ is a function of the design decision variables and the uncertain parameters. The original variable vector z is split into two vectors, x and a, such that vector x is the decision variables and vector a is the uncertain parameters; this allows the parameters and variables to be handled differently.

Generalized Problem:

$$\begin{aligned} min \quad & f(z) \\ s.t. \quad & g_i(z) \le 0 \\ & h_i(z) = 0 \\ & z \in Z \end{aligned}$$

Variable vector is decomposed:

$$z = x + a$$

x = decision variable vector
a = uncertain parameter vector

Acknowledgements

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In early stage design there is a low level of design definition leading to high levels of uncertainty surrounding decisions

A low level of design definition results in unknown parameter values and unknown requirements, both of which are needed to fully evaluate designs to make design decisions. Current approaches to handle uncertainty in early stage design include robust optimization (RO), reliability-based design optimization (RBDO), and set based design (SBD).

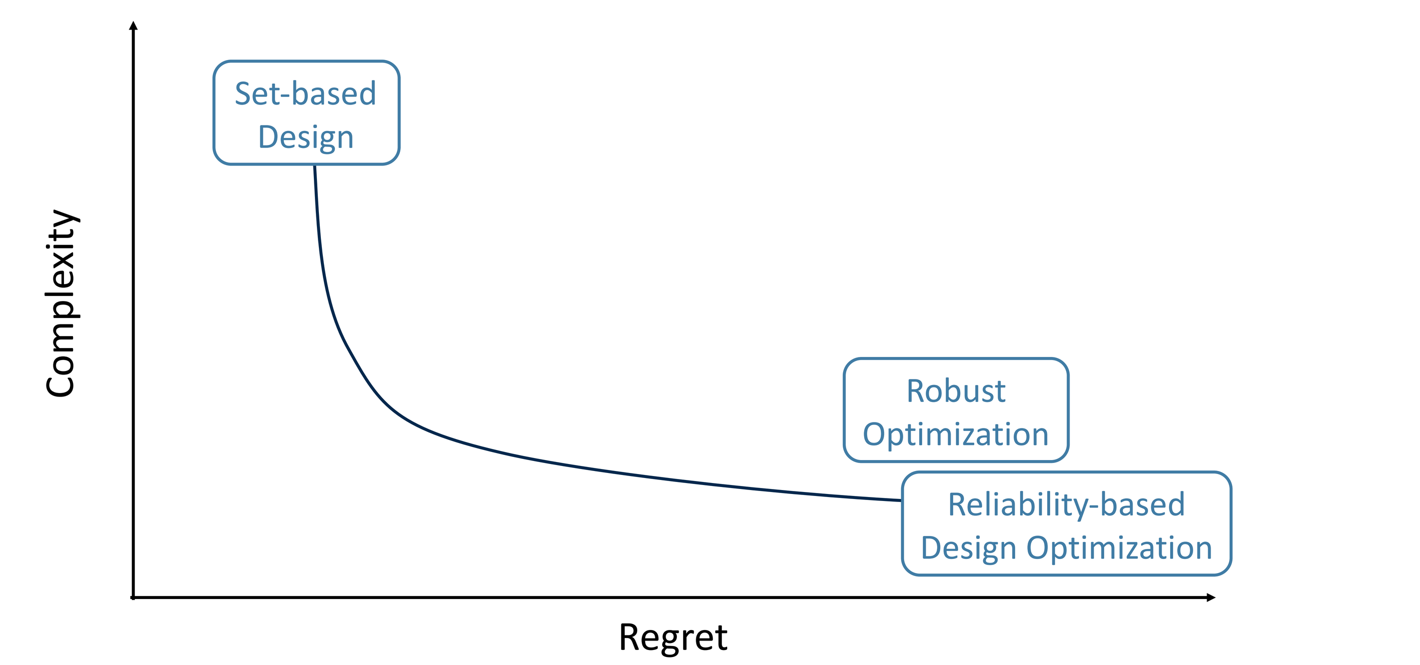

Figure 1: Existing methods to handle uncertainty in early stage design evaluated in terms of complexity and regret. There are no existing methods in the center of the complexity-regret trade-space where compromises are made between complexity and regret.

How can the trade space between complexity and regret be explored in early stage design?

A method that has two optimizers was used to explore the complexity-regret trade space for small structural problems

A secondary optimization problem is contained in the objective function calculation for the main multi-objective optimization problem. The main, or outer, optimization problem is solved using a genetic algorithm (GA) and has complexity and regret for the objectives. Each individual of the GA describes a sub-space of the original design space where the secondary, or inner, optimization problem is used to evaluate the complexity and regret of the subspace. The secondary, or inner, optimization problem evaluates the design fitness and is solved for each possible realization of the uncertain parameter giving a set of optimal solutions in the space; this set is used to evaluate the complexity and regret.

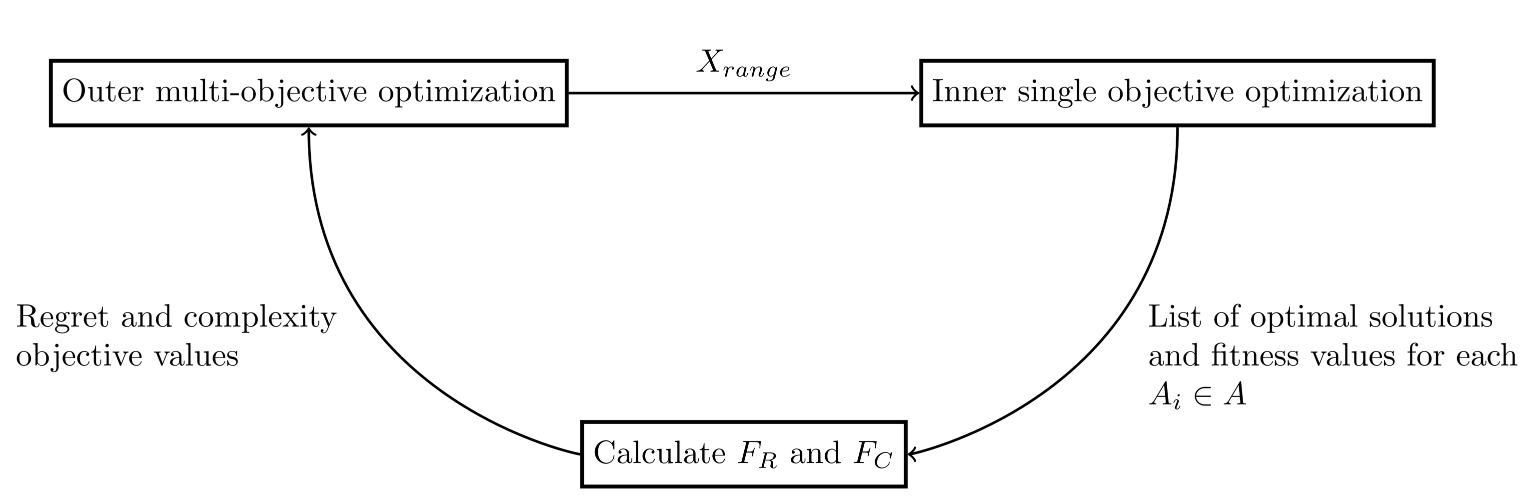

Figure 4: The method structure is shown. The outer optimization problem gives an X_{range} to the inner optimization problem which provides a set of optimal solutions for the uncertainty set which is then used to evaluate the complexity and regret of the subspace X_{range}.

The method was used to explore the complexity-regret trade space for the small design problem of a t-stiffened box girder

The design of a t-stiffened box girder that resembles a ship hull girder was explored using the presented method. The problem had six design variables (plate thicknesses, stiffener sizes, and stiffener spacing), and one uncertain parameter (required bending moment). The design variables and uncertain parameter were all discrete variables with eight possible values. The box girder designs were evaluated to minimize material and production costs and weight.

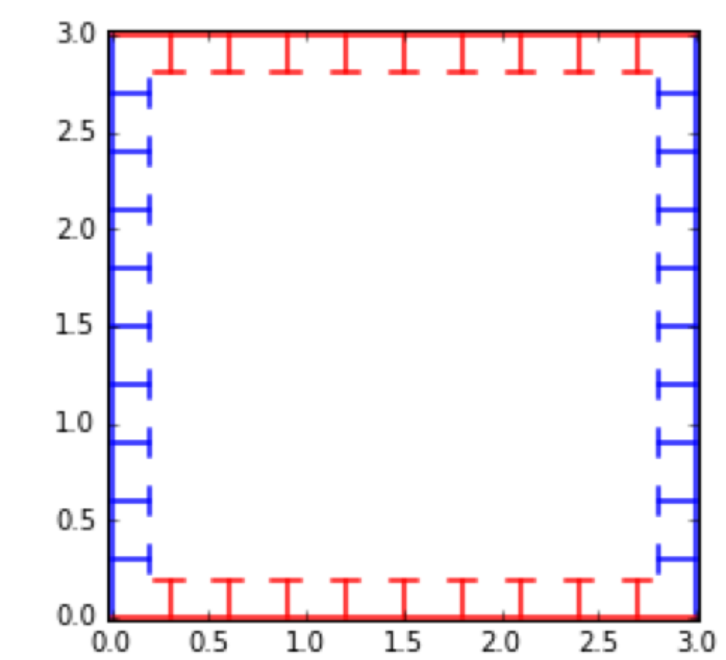

Figure 5: The complexity-regret trade space for the design of the shown t-stiffened box girder was explored

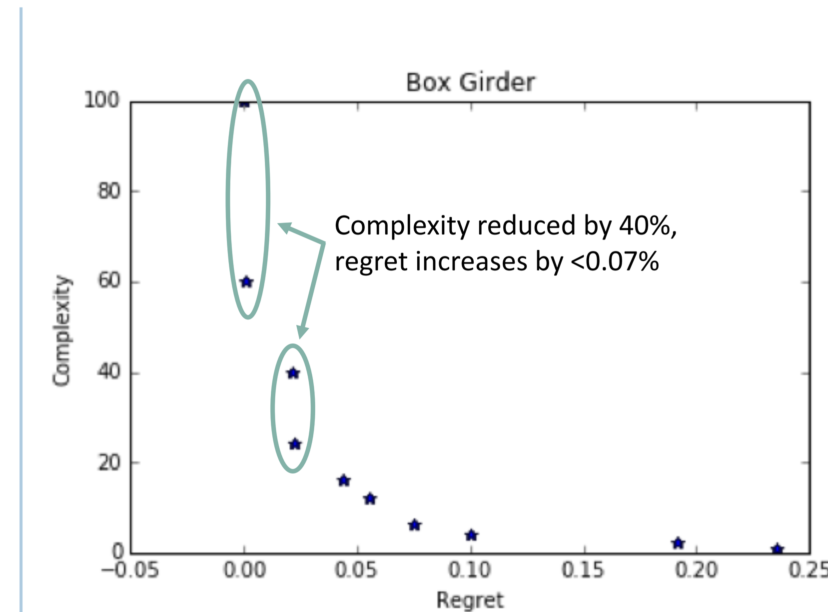

Figure 6: The Pareto front in the complexity-regret trade space for the design of a t-stiffened box girder is shown. The Pareto front is sparse due to the discrete variables in the problem .

The method was used to explore the complexity-regret trade space for the small design problem of a cantilever tube

The design of cantilever tube was explored using the presented method. The problem had two design variables (average tube diameter, and average tube thickness), and two uncertain parameters (angles of the two applied forces). The cantilever tube design fitness was evaluated as the cross sectional area of the tube.

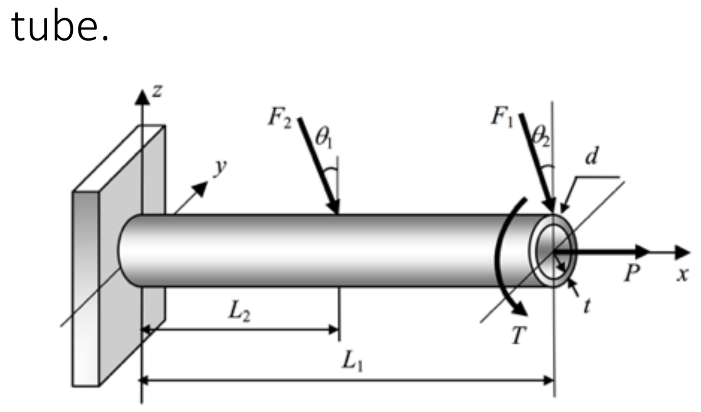

Figure 7: The complexity-regret trade space for the design of the shown t-stiffened box girder was explored. The average diameter, $\boldsymbol{d}$, and thickness, $\boldsymbol{t}$, were the design variables, and the angles of the applied forces, θ_1 and θ_2, were the uncertain parameters.
Figure reproduced from: Du, X., 2007. Interval Reliability Analysis. 33rd Design Automation Conference, Parts A and B, 6(3), pp. 1103-1109

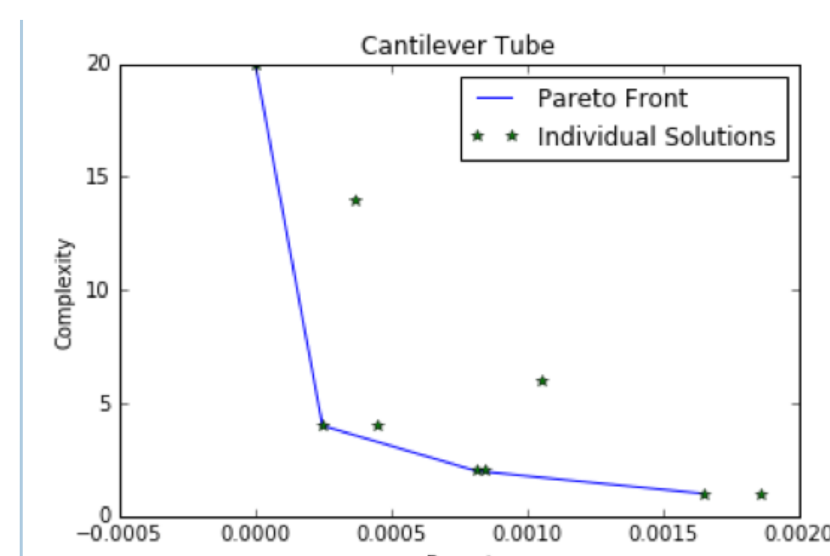

Figure 8: The Pareto front in the complexity-regret trade space for the design of a cantilever tube is shown. The resulting Pareto front is sparse due to the small space and discrete variables.

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